AMERICAN SOCIETY OF CIVIL ENGINEERS.

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CLX.

(Vol. VII.—May, 1878.)

ON THE CAUSE OF THE MAXIMUM VELOCITY OF WATER FLOWING IN OPEN CHANNELS BEING BELOW THE SURFACE.

By JAMES B. FRANCIS, C. E., Member of the Society.

Hydraulic engineers have been very unwilling to admit that, generally, in open channels the greatest velocity is not at the surface, but at some distance below; the fact is, however, now too well established by numerous experiments to be any longer doubtful. Some have attributed it to the resistance of the air, and this, undoubtedly, must frequently tend to produce such an effect, more particularly when the wind is strongly opposed to the current; but it has been observed that the greatest velocity is at some distance below the surface when the wind is in the same direction as the current, when its tendency must be to quicken the surface velocity.

At Lowell, Massachusetts, the water drawn from the canals by the manufacturing companies, is in several cases, gauged in what are there

called *measuring flumes*. These are constructed in parts of the canals where the section is rectangular and the alignment as free from bends or other irregularities as practicable; they are simply linings of the bottom and sides with smooth planking, retaining the direction, and, as nearly as practicable, the dimensions and form of the canal. The velocity of the flow in these flumes is ascertained by means of tin tubes, loaded at one end with lead, so as to sink nearly to the bottom. It is observed that there is a continual change in the velocity in all parts of these channels, although there may be no sensible change in the volume of water flowing, and, consequently, none in the *mean* velocity; this must result from a continual change in the position of the currents moving with different velocities.

The contact of the water with the planking forming the sides of the flume retards the velocity of the adjacent parts of the stream; parts more distant from the sides move more rapidly; this difference in velocity tends to produce eddies in which the water is, of course, moving in all directions. One effect of this, I think, is to allow some of the quicker moving currents to penetrate those moving more slowly, and to detach masses of water from the slow currents, and force them further from the sides, where they become the slower currents among those moving quicker.

To ascertain the velocity in different parts of the width of the flume, the tubes are put into the water, and, when in a vertical position, are abandoned to the current, and the time of their passage between two cross sections observed; from this their mean velocity is obtained. To test this method of gauging the flow of water, experiments were made in a flume fitted up for the purpose, in which the usual causes of irregular motions were reduced to a minimum. The arrangements for ascertaining the velocities of the tubes by means of an electric telegraph and a marine chronometer, operated by skillful observers, also reduced the errors of observation to a minimum.

The variations in the velocity in particular parts of the flume when there was no sensible change in the mean velocity, were, however, very marked. This is plainly shown by the diagrams in Plate XVII., Lowell Hydraulic Experiments,* in which the velocities in several observations are represented by the ordinates of the small circles and the mean velocity by the corresponding ordinate of the full line. The

* Lowell Hydraulic Experiments, 2d Edition. D. Van Nostrand. New York. 1868.

variations in the ordinates of the small circles for the same abscissa, indicate the changes of velocity at that part of the flume, which result from the changes in the currents.

In the diagram of Experiment 1 the variations extend from 8.57 per cent. above the mean to 11.40 per cent. below ; the average variation in thirty observations in different parts of the width of the flume being about 3.12 per cent. These variations are deduced from the mean velocities of the tubes through a course of seventy feet ; the variations in some parts of the course must have largely exceeded these averages. The variations in velocity just described are deduced from exact observations in different parts of the width of the stream ; similar, and probably greater, variations must occur in different parts of the depth, resulting from the same cause, namely, the constant interchange in the position of the currents moving with different velocities ; we have not, however, the observations at hand to verify it, and they would not be so readily made ; but we may safely infer from those already described, that they do exist. In rectangular channels in which the depth is less than half the width, the retarding effect of the bottom must exceed that of the sides, and the eddies and other irregular motions resulting, must be greater in the same ratio, and the effect previously described, of the quicker currents penetrating those moving slower, and detaching, and, in this case, forcing upwards, masses of the slow moving water, must occur to a greater extent than results from the retardation caused by the sides.

These slower-moving masses would be carried along by the quicker moving water, and would soon, but not at once, acquire the same velocity; many of these would reach the surface before fully acquiring the normal velocity, where their living force, in a vertical direction, would be expended in spreading out laterally, and where the force was considerable, in producing the boils so frequently seen, particularly in natural water courses.

The constant recurrence of the same phenomena, must cause an accumulation of water at and near the surface, moving with a mean velocity somewhat less than the mean velocity of the water at some distance below the surface. If these views are correct, there must be a constant movement of water from near the bottom of an open channel of running water to the surface; to test this, the following experiments were made in the canals at Lowell, supplying the manufacturing establishments with water for motive power.

EXPERIMENTS IN THE WESTERN CANAL, JULY 6, 1867.

These experiments were made in a straight reach of the canal, about 1 200 feet in length, in which the section was nearly uniform, viz.: about 20 feet wide at bottom, 50 feet wide at the surface of the water, and about 10 feet deep. The velocity of the water in the middle of the canal at the surface, was about 3.4 feet per second. No water is added to or abstracted from the canal in this reach. This canal is excavated partly in gravel and partly in rock; the bottom of the canal is now all gravel, with a few small stones which have rolled in from the sides; a portion of the sloping sides are rock; much the longest portion, however, in the part of the reach where the experiments were made, is gravel, having only moderate inequalities of surface. A large flat-bottomed boat, drawing a few inches of water, was moored about 500 feet from the up-stream end of the reach. On one side of the boat, in the middle of the canal, a lead pipe of two inches interior diameter, attached to a small post, was placed vertically, with the lower end bent, so that the centre of the lower orifice was about five inches above the bottom of the canal, and pointed down stream, in the direction of the current. The pipe extended about four feet above the surface of the water, and was connected, at its upper end, to a tub which contained about 3.1 cubic feet of whitewash. In the bottom of the tub was an orifice, communicating with the pipe, which could be closed by a plug.

At the commencement of an experiment the tub was nearly filled with whitewash, at a given signal the plug was withdrawn, the whitewash was discharged at the lower orifice in about 22″; observers were stationed on the banks below, to note the appearance of the whitewash at the surface. The water was nearly clear, permitting the banks to be seen at a depth of three or four feet.

Eight trials were made, in which the whitewash appeared at, or very near, the surface, at distances from the lower orifice in the pipe varying from about 100 to 175 feet; the mean of the distances, as recorded, being 129 feet.

EXPERIMENTS IN THE NORTHERN CANAL, JULY 25, 1867.

These experiments were made in a reach of the canal of uniform section, straight, free from obstructions, and about 1 100 feet in length, in which no water was added or abstracted. The canal has walled sides, and is 97 feet wide at the bottom and 103 feet at the surface of the water, which is 16.5 feet in depth; a small portion of the excavation is in rock; the bottom of the canal is nearly all gravel, with irregularities of eleva-

tion of only a few inches. The sides are walled, except at a few points where the rock projects, in no place, however, more than 1.5 feet from the line of the face of the wall. The velocity of the water in the middle of the canal, at the surface, was about 2.2 feet per second.

An examination of the bottom of the canal was made a few days before the experiments were made, the water having been drawn off; at this time all the stones on the bottom exceeding about three inches in their smallest dimension, were removed from the central parts of the canal and placed near the sides. The flat-bottomed boat, used in the western canal, was moored near the middle of the reach, and arrangements made for discharging whitewash near the bottom of the canal, similar to those in the western canal, previously described, the pipe being of wood, four inches square inside, to the top of which was attached a tub holding 5.56 cubic feet of whitewash, which was discharged on the removal of a plug, in about four seconds.

Five trials were made, in which the whitewash appeared at, or near, the surface, at distances varying from about 300 to 500 feet from the point at which it was discharged into the canal. The sides of the canal are but little above the level of the surface, which, combined with the width of the canal, did not permit the observations to be made so definitely as in the western canal, where the banks are higher and the canal about one-half the width; about 530 feet below the point where the whitewash was discharged a bridge crosses the canal, from which the whitewash was seen at or very near the surface on the up-stream side of the bridge, in each of the five trials. In one trial only the whitewash was observed at about 300 feet from the point where it was discharged. In one trial the whitewash was first seen near the bridge in 4′ 55″ after it was discharged, and masses of it continued to be seen at short intervals for about 1′ 30″. In another trial it was first seen near the bridge in 4′ 58″ after it was discharged and continued to be seen for about 1′ 20″. The whitewash, as it appeared near the surface, approaching the bridge, was much diffused, but quite distinct, and in masses resembling the globular masses of smoke discharged from chimneys, at some distance from the tops of the chimneys.

From these experiments it appears that the water at the bottom of the streams came to the surface, at distances varying from about ten to thirty times the depth, the shorter distances being in the canal of the least depth and most uneven bed.

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CLXI.

(Vol. VII.—May, 1878.)

THE FLOW OF WATER IN PIPES UNDER PRESSURE.

By CHARLES G. DARRACH, C. E., Member of the Society.

In the well known equation for the flow of water in pipes,

$$h'' = \frac{CLmv^2}{2gS} \text{ or } h'' = \frac{4mLv^2}{2gd} *$$

the value of "m" has been given by various experimenters for straight clean pipes of different diameters, and for different velocities.

The following tables have been deduced from experiments on the pumping mains of the Philadelphia Water Works, and show the effect produced by abrupt bends, check valves and foul pipe.

The temperature of the water was about 62°, and its weight has been considered as 62.338 lbs. per cubic foot.

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* h'' = friction head in feet.
v = velocity of flow in feet per second.
g = 32 feet.
m = coefficient of flow.
$\frac{S}{C} = \frac{d}{4}$ = hydraulic mean radius in feet.
C = contour of pipe in feet.
S = sectional area.
L = length of pipe in feet.
d = diameter of pipe in feet.

Experiment No. 1.—May 23, 1878.

Belmont Works—30-in. main, laid in 1869. Engine, "Worthington." 5 per cent. deducted for slip. One curve, of short radius. Gauge on main outside check valve.

$$m = \frac{2gh''d}{4Lv^2}$$

30 inch Main, laid Nine Years.

ENGINE.		PIPE.		Velocity in Feet per Second.	HEAD.				Length of Pipe.	m.
Revolutions per Minute.	Cubic Feet per Minute.	Diam.	Area.		Gauge.	Total Head.	Static Head.	Friction $H-h$.		
		d.	S.	v.	lbs.	H.	h.	h''	Ft.	
0				0	82.25	190	190	0.		
8				1.070	84.00	194.04	"	4.04		.03208
9	357.3			1.205	84.25		"	4.62		.02900
10	397.0			1.340	84.50	195.2	"	5.20		.02633
11	436.7			1.475	84.75		"	5.78		.02416
12	476.4	30" = 2½ ft.	4.91 sq. ft.	1.610	85.00	196.35	"	6.35	4,400	.02229
13	516.1			1.745	85.25		"	6.93		.02069
14	555.8			1.880	85.50	197.51	"	7.51		.01931
15	595.5			2.015	85.75		"	8.09		.01811
16	635.2			2.150	86.00	198.66	"	8.66		.01703
17				2.285	86.25		"	9.24		.01609

EXPERIMENT NO. 2.—JUNE 1, 1878.

Spring Garden Works—36-in. main, laid 1871. Engine, "Simpson." All curves, 25 feet radius; no checks.

$$m = \frac{2gh''d}{4Lv^2}$$

36-INCH MAIN, SEVEN YEARS OLD.

ENGINE.		PIPE.		Velocity in Feet per Second.	HEAD.				Length.	m.
Revolutions per Minute.	Cubic Feet per Minute.	Diam.	Area.		Gauge.	Total Head.	Static Head.	Friction Head $H-h$.		
		d.	S.	v.	lbs.	H.	h.	h''	Ft.	
					43.25	100.				
10	670			1.58	44.78	103.45		3.45		.01791
11	737			1.74	45.03	104.03		4.03		.01718
12	804	36 in.=3 ft.	7.07 sq. ft.	1.89			100 ft.	4.61	3760	.01673
13	871			2.05				5.19		.01601
14				2.21				5.77		.01536
15				2.37				6.35		.01466

EXPERIMENT No. 3.—JUNE 1, 1878.

Roxborough Works. 20" main—laid in 1867. Engine—"Worthington." 5 per cent. deducted for slip. One quarter turn at basin. Gauge on pump.

$$m = \frac{2gh'd}{4Le^2}$$

20-INCH MAIN, LAID ELEVEN YEARS.

ENGINE.		PIPE.			HEAD.					
Revolutions per Minute.	Cubic Feet per Minute.	Diam.	Area.	Velocity in Feet per Second.	Gauge.	Total Head.	Static Head.	Friction $H-h$.	Length of Pipe.	m.
		d.	S.	e.	lbs.	H.	h.	h'.	Ft.	
0					135.5	313.0	313.0	0		
9				2.71	142.75		"	16.75		.01419
10	394			3.01	144.	332.64	"	19.64		.01348
11	433.4			3.31	145.25		"	22.52		.01269
12	472.8	20 ins. = $1\frac{2}{3}$ feet.	2.18	3.61	146.50	338.42	"	25.42	4320	.01204
13	512.2			3.91	147.75		"	28.31		.01147
14	551.6			4.21	149.0	344.19	"	31.19		.01093
15	591.0			4.51	150.25		"	34.08		.01033
16	630.4			4.81	151.5		"	36.96		.00986
				5.11	152.75		"	39.84		.00941

EXPERIMENT No. 4.—JUNE 5, 1878.

Delaware Works. 36″ main. Laid in 1871. Engine—"Worthington." 5 per cent. deduction for slip. Gauge on stand pipe. Curves in pipe made with 25 feet radius, except at the basin where 90° are made with a T pipe.

$$m = \frac{2gh''d}{4\,Lv^2}$$

36-INCH MAIN, LAID SEVEN YEARS.

ENGINE.		PIPE.			HEAD.					
Revolutions per Minute.	Cubic Feet per minute.	Diam.	Area.	Velocity in Feet per Second.	Gauge.	Total Head.	Static Head.	Friction Head $H-h$.	Length of Pipe.	m.
		d.	S.	v.	*lbs.*	H.	h.	h''	*Ft.*	
0					51.25	118.4	118.4	0		
9	423			1.000				13.00		.05033
10	470			1.111^1				13.28		.04170
11	517			1.222^2				13.57		.03506
12	564			$1\frac{1}{3}$	57.25	132.25		13.86		.03021
13	611	36 ins. = 3 feet.	7.07 square feet.	1.444^4				14.15	12.400	.02626
14	658			1.555^5	57.50	132.83		14.44		.02314
15	705			$1\frac{2}{3}$				14.73		.02055
16	852			1.777^7	57.75	133.40		15.01		.01847
17				1.888^8				15.30		.01667
18				2.000				15.59		.01509

EXPERIMENT No. 5.—JUNE 6, 1878.

Frankford Works. 30" main laid 1876. Engine "Cramp." 5 per cent. deducted for slip. Grade undulating.

Curves of 25 feet radius.

4 check valves on main.

Area of openings in check, 750 sq. inches.

Weight of valve, 320 lbs. in 1 check.

Gauge on air vessel. Weight of 4 checks. 1 280 lbs.

$$m = \frac{2gh''d}{4Lv^2}$$

30-INCH MAIN TWO YEARS OLD.

ENGINE.		PIPE.			HEAD.					
Revolutions per Minute.	Cubic Feet per Minute.	Diam.	Area.	Velocity in Feet per Second.	Gauge.	Total Head.	Static Head.	Friction Head $H-h$.	Length of Pipe.	m.
		d	S.	v.	lbs.	H.	h.	h''	Ft.	
0						167.3	167.3			
10	433			1.47				10.43		.00947
11	476.3			1.62	77.5	179.03		11.73		.00885
12	519.6			1.76	78.+			13.03		.00833
13	562.9			1.91	78½+			14.33		.00777
14	606.2			2.06	79¼—			15.63		.00726
15	649.5	30 ins. = 2.5 feet.	4.91 square feet.	2.20	79.75	184.23		16.93	20,200	.00692
16	692.8			2.35	80¼+			18.23		.00653
17	736.1			2.50	80¾+			19.53		.00618
18	779.4			2.64	81½—			20.83		.00591
19	822.7			2.79	82.	189.42		22.12		.00562
20	866.0			2.94	82½+			23.42		.00534
21	909.3			3.08	83⅛+			24.72		.00516
22	952.6			3.23	83.7			26.02		.00493

Deduction From Experiment No. 5.

Frankford 30" Main.—The check-valves in the main produce a pressure of 1.8 lbs. per square inch, which is equal to a head of 4.16 feet. This amount deducted from the friction head in Table No. 5 gives the corrected friction head :

$$m = \frac{2gh''d}{4Lv^2}$$

30-inch Main. Two Years Old.

Engine.		Pipe.			Head.					
Revolutions per Minute.	Cubic Feet per Minute.	Diam.	Area.	Velocity in Feet per Second.	Gauge.	Total Head.	Static Head.	Friction Head. $H-h$.	Length.	m.
		d.	S.	v.	lbs.	H.	h.	h''.	Feet.	
10	433			1.47		173.57	167.3	6.27		.00575
11	476.3			1.62	75.7			7.57		.00571
12	519.6			1.76				8.87		.00561
13	562.9			1.91				10.17		00552
14	606.2			2.06				11.47		.00535
15	649.5			2.20	77.95	180.07		12.77		.00522
16	692.8	30" = 2.5 ft.	4.91 sq. ft.	2.35				14.07	20,200	.00503
17	736.1			2.50				15.37		.00487
18	779.4			2.64				16.67		.00473
19	822.7			2.79	80.2	185.26		17.96		.00452
20	866.0			2.94				19.26		.00441
21	909.3			3.08				20.56		.00429
22	952.6			3.23				21.86		.00415

EXPERIMENT No. 6.—JUNE 11, 1878.

Roxborough 30-inch Main laid 1878.—Engine, "Worthington." 5 per cent. deducted for slip. One quarter turn. All other turns made with curve pipe of 25 feet radius. Two check-valves on main.

The weight of the valves, 1.3 lbs. per square inch, has been deducted from the observed pressure on the gauge.

$$m = \frac{2gh'd}{4Lv^2}$$

30-INCH MAIN. NEW.

ENGINE.		PIPE.			HEAD.					
Revolutions per Minute.	Cubic Feet per Minute.	Diam.	Area.	Velocity in Feet per Second.	Gauge.	Total Head.	Static Head.	Friction Head. $H-h$.	Length.	m.
		d.	S.	v^2.	*lbs.*	H.	h.	h'.	*Feet.*	
0					140.33	324.16		0		
12	472.8			1.60	141.0	325.71		1.55		.00606
14	512.2			1.74				1.83		.00604
14	551.6	30 inches = 2.5 ft.	4.91 sq. ft.	1.87	141.25	326.27	324.16	2.11	4000	.00603
15	591.0			2.00				2.40		.00600
16	630.4			2 14	141.5	326.84		2.68		.00590

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(Vol. VII.—May, 1878.)

DISCUSSIONS ON

THE CAUSE OF THE MAXIMUM VELOCITY OF WATER FLOWING IN OPEN CHANNELS BEING BELOW THE SURFACE, AND ALSO ON THE FLOW OF WATER IN PIPES UNDER PRESSURE.*

By THEODORE G. ELLIS, CHARLES E. EMERY, CLEMENS HERSCHEL, DE VOLSON WOOD and J. T. FANNING.

THEODORE G. ELLIS.—I think Mr. Francis is undoubtedly right in saying that it is not the effect of wind or the resistance of the air upon the surface of the water that causes the retardation, as numerous experiments have proved that when the wind is blowing down stream the water will still be retarded at the surface. It is likewise true that the currents are all the time being interlocked and mixed. The thread of greatest velocity is never long in the same place; it is constantly swaying from side to side and rising and falling. But I cannot readily understand Mr. Francis' idea of the coming up of more slowly moving masses of matter from the resistance of the bottom—in the shape of a curve. From observations upon the Mississippi and other rivers it would appear that if the resistance comes from the bottom and acts

* Referring to Papers CLX and CLXI, Vol. VII (May, 1878).

directly upwards, every particle of water that moves from the bottom must pass through the thread of greatest velocity. I do not see any good reason why the greatest velocity of this thread should not be given to every particle of water that passes up through it. If the retardation is communicated from the bottom to the top, it must be in curves, so that the rising water must have a retrograde movement as it passes above the thread of greatest velocity before it gets to the top. It cannot, it seems to me, be transmitted directly.

It has troubled hydraulic engineers a great deal to know what causes this resistance at the surface, and I will confess that I am unable to solve that problem, although there is some reason, in my opinion, to believe that it is caused by the resistance of the banks and of the bottom near the banks. We find that in very wide and shallow streams the velocity approaches very near the surface, while in deep streams, with steep banks, the thread of greatest velocity is carried lower down. As instances of this there are the experiments of Mr. Révy, on a section five miles wide with a depth of forty or fifty feet, in which he observes the current and finds the greatest velocity at the surface; and in the experiments of Darcy and Bazin, as published in their work, we find that in rectangular channels we have the greatest velocity nearly in the middle of the section: the retardation at the middle of the surface seems to be communicated in some way from the sides, the water near the sides being so much slower, would seem, being near the surface, to hold it back near the middle. It seems to me that this offers the only plausible solution of the problem. It is found that in very wide rivers we have the greatest velocity near the surface, and in rivers with contracted and steep, perpendicular banks, the thread of greatest velocity is carried down almost to the middle. That fact would seem to verify this theory.

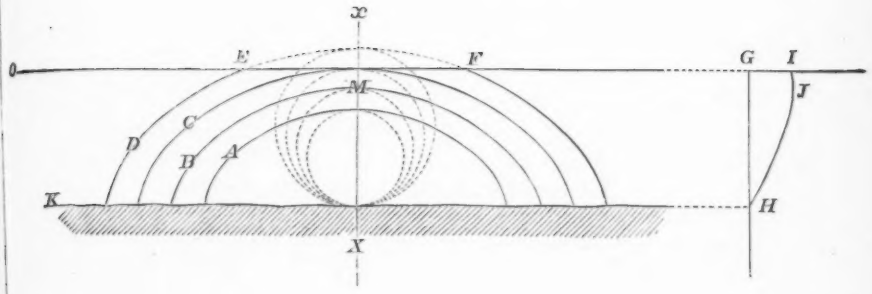
There is another peculiarity about the curve of vertical velocities to which my attention was first called by Gen. Warren of the U. S. Engineer Corps. The resulting curve from a large number of experiments always has a depression just below the middle, almost making it a straight line. It is flattened so as to make the general curve to near the bottom a parabola. In trying experiments nearer the bottom than were tried on the Mississippi, we find that the curve recedes very rapidly. There is a flat place left between the bottom and middle. This is a feature of all the vertical curves that I have observed, and it is quite marked in some of them. So that it seems that there are really two

systems of forces acting, one to produce the curve at the bottom and another to produce the curve at the top.

In reference to the flow of water in channels, it will require a great deal more observation than has yet been given to it to find laws that will govern any number of streams. Formulæ that are applicable to the Mississippi and large rivers do not appear to be applicable to small rivers, and formulæ that are applicable to small streams are not applicable to large ones.

CHARLES E. EMERY.—While listening to the paper of Mr. Francis and the remarks of Gen. Ellis, on this subject, it has occurred to me that the phenomena can be explained by considering that the combined vertical and progressive movements of the water resemble, on a large scale, the eddy resistance caused by the passage of fluid along surfaces in general. It has been demonstrated that each particle of water in a wave moves in a circle, and that the several particles following each other in order cause the successive crests and hollows together with an appearance of progression which does not actually exist. In eddy resistance, however, the particles move in a circle which has a progressive movement, so that the path of each particle in space is in a cycloid or a modification of the same.

To apply this principle to the motion of water in open channels, we have but to consider that the particles of water are continually receiving impulses upward (or normal to the bed of the stream) by deflection of the current from projections on the bottom or the fluid retarded thereby. The upward motion combines with the progressive movement of the current, but the particles continue to rise until stopped by forces opposite and equal to the original ones. Such opposite or downward forces result from the necessity of continually re-supplying the water projected upward. The path moved through by each particle should, therefore, resemble in its general features a cycloid,—at least so far as repetition of the curve is concerned,—though, perhaps each branch may be modified or distorted so, for instance, as to resemble the path of a projecticle. The distances above the bottom reached by the different particles would vary with the intensity of the deflecting forces, so that motion should result in a number of somewhat similar curves of different amplitude with their ascending and descending portions continuously crossing each other.



Representing the approximate paths of a few particles by the cycloidal curves *A B C D*, the ordinates will correspond to the variations in velocity. The reduced velocity of the surface may be accounted for by the fact that the upward impulse of some particles will be sufficient to carry the corresponding curve above the surface *O*, *G*, of the stream, when, necessarily, such particles spread out on the surface before attaining the maximum velocity at *x*. At some point below the surface, *M* for instance, the curves are uninterrupted, and the maximum velocity of the section is obtained. Representing the velocity at different heights by the ordinates of a curve, *I H*, measured from a vertical *G H*, it should be expected, from the above considerations, that the maximum would show below the surface at about *J*, as has been found practically and stated by the previous speakers. It is evident, also, that as the bed of the stream is deepened and approximates more nearly a semi-cylinder, the several curves normal to the surface of the bed should unite to raise the maximum velocity nearer to the surface, as has been stated by Gen. Ellis.

Clemens Herschel. – This touches upon a subject that forms a familiar theme of study with me, and has formed one for many years. And perhaps it will be well if I take the occasion to make what I shall call an appeal to the engineers present that they give us some better data upon this subject than we have yet been able to obtain. I should like to have this element introduced into this investigation—the element of

simultaneous observations. When we speak of a velocity curve—a curve of any shape you please—the one drawn on the black board or any other, and of any point on that curve—plotted as any intelligent engineer would plot a curve, the method which was followed to get that point or curve was this : the engineer put an instrument (of whatever sort you please), at that point to-day, at such an hour and minute, and he put it there ten minutes afterwards, and perhaps half an hour afterwards, and perhaps a week afterwards, and, from a great many times doing this, he says that in that cross section, and for that point, the average is there ; and then, during a different series of times the same thing is done in another place and the result given is that the average is *that*. Now, I have been so fortunate as to have done a good deal of work in an establishment that we shall visit by and by, where this principle of *simultaneous observation* has been carried on for a great many years, and has produced results of great value in hydraulic engineering. And it seems to me that if this same principle of simultaneous observation be applied here, it will produce equally novel and valuable results. I hope some engineer, who is so situated that he can do it, will carry out the suggestion. I want him to use a vertical wire or rod, or something of that sort, put down in the river, and I want half a dozen or more current meters put on that wire and connected with registers at the surface, by the use of electricity, so that the record of them all can be read simultaneously, and I want that record kept so that we can avail ourselves of it during the continuance of any reasonable length of time. I want these meters strung along at different depths and also at different places transversely, and I think that with a series of such observations we shall very soon arrive at that state of knowledge which we all are so much wishing for, and upon which alone safe theories could be founded.

De Volson Wood.—The remarks made by Mr. Francis in regard to the intermingling of the threads of water—perhaps I might say the particles of water—I think will commend themselves with some force to all of us. I will not presume to give a satisfactory reason for the thread of maximum velocity being at any particular point below the surface. But I will venture to make a suggestion ; I would not presume to make any statement in regard to it which would be construed into a theory, because so many others have thought upon the same subject before. It is said that particles from the bed of a stream work their

way up, so to speak, to the surface, and it is evident that if that takes place the particles at the surface find their way down to the bed. Now, we know that when a particle is acted upon by gravity, and is free to move, the acceleration of the velocity increases as the particle descends, and it seems to me that there may be some force in the presentation of it in this case. Admit that the particles ascend. They begin with a very slow velocity, and are constantly coming in contact with those which have a greater velocity, and these act upon each other, the slower retarding the faster, and the faster urging on the slower; and this conflict goes on until a condition of equipoise is reached. If that be the only operation, we must necessarily conclude, I think, that the maximum velocity would be at the surface. But those particles which have now reached the surface, must find their way back to the bottom again, and when they descend they will be retarded by passing the particles below, but accelerated by gravity, and from these causes may gain a point of maximum velocity somewhat below the surface. I do not see any particular force in the suggestion that these particles may move in cycloidal arcs, for I think it is extremely doubtful if they do. The subject of vortex motion has been raised in the question, and perhaps many of my hearers know quite as well as I, or even better, that Professor Reynolds, of Manchester, England, made some experiments some two years since, in regard to vortex motion of water in a trough, by introducing some colored matter into the water, by means of which he could see the peculiar motions which took place. A rapid circular motion of small portions of the liquid was produced, which passed rapidly the length of the trough without apparently disturbing the surrounding water. This motion is sometimes seen in the whirling motion of the ring of smoke as it ascends from a pipe or cigar, and passes through the air without involving that which lies about it. We also see that the small whirlpools in streams move rapidly through the water without disturbing the surrounding water. Now, when we recognize the fact that these complex motions do exist in flowing streams, I think it is not strange that a formula deduced from the flowage of one stream will not apply to others.

This brings to mind a remark that I have often made in regard to hydraulic engineers. There appears to be a peculiar satisfaction with them in discarding all that has been done before, and find fault with all their predecessors, and especially with those who have written on the

subject. The latest theorist believes that he has passed over and beyond the errors of those who preceded him. At least it appears as though the subject had not been, and perhaps cannot be, reduced to that exact scientific certainty which will prevent other investigators from modifying existing formulæ. The varied circumstances under which we see bodies of flowing water—in deep streams, narrow streams, large streams, and streams of all degrees of velocity—makes it quite evident that we cannot expect to arrive at a certain mathematical law. It is one of those subjects which seem to be beyond the grasp of pure analysis, and, consequently, is necessarily left to the hands of the experimenter. Everything which promotes accuracy of observation tends towards a more correct solution of the problem. Hence, Mr. Herschel's remark in regard to simultaneous observations was a very valuable one, and ought to be carried out. I suppose we can hardly doubt that the curve of velocity noted by Gen. Ellis and Mr. Francis is sufficiently accurate to establish the fact that the maximum velocity is generally greater at a point somewhat below the surface than at the surface. It is a matter that we would like to explain on theoretical grounds. But let us have the facts first, then the theory.

Charles E. Emery.—The remarks of the last speaker, that "the water may move in cycloidal curves, but that it is doubtful if it does," are hardly warranted in view of investigations that have already been made on the general subject. I call to mind, particularly, that the skin resistance of vessels has been satisfactorily explained on the hypothesis that the water rolls in circles on the surface as the vessel progresses, so, evidently, the motion of each particle is in a cycloid. The conditions in the case under consideration are entirely similar. It only requires that the deflecting forces be sufficient to cause the particles in due time to reach the surface, when, as has been shown, the various interesting phenomena described by former speakers are readily accounted for.

J. T. Fanning.—I had not expected to say anything upon this topic.* It is one very closely connected with what Mr. Francis and others have been discussing. As Mr. Darrach has presented the subject, it relates to the flow of water *under pressure*, while, as presented by Mr. Francis, it relates to the flow of water with a *free surface*, as in a stream or canal, and to the resulting currents.

* Referring to Paper CLXI, Vol. VII (May, 1878).

If we consider the motion in this channel, sketched here upon the black board, which has, say sixteen feet depth, as that of a body of molecules of water, and think of the depth as in molecules, a thousand or more millions of molecules in depth, all of which move individually, and move past and between each other with very little friction, then, perhaps, we can more clearly understand the movement of the water currents or molecule groups.

Now, for instance, we will suppose that the minute individual particles of water in this channel are moving forward with a velocity relatively due to the sine of the angle of inclination of the water surface; thus, if there is one foot inclination in a mile, then there is a velocity which all these particles tend to take which is due to that inclination.

As these particles move forward they encounter obstructions, like, for example, this piece of rock sketched on the bed of the stream, then these particles that come in contact (and the group immediately surrounding them) are deflected. A different direction is given to them from that which they tended to take flowing under the influence of gravity, and the deflected group of particles passes through and between the particles moving in the first direction. Consequently, having taken a new direction, and having their progress checked by reaction from the obstruction, their *onward* progress is less than that of the main body of water, and the *mean* velocity of the whole body is reduced. Also, if the deflected group of particles passes up through the main body and reaches the surface, it is there unable to continue in the new direction, and is again deflected in part with a spreading movement towards the sides of the channel, with a further loss of onward velocity.

Considering the motion of the particles in this manner, we can readily understand how the current changes its velocity in certain parts, at the bottom, at the sides, and at the surface, as is demonstrated by experiment.

Those groups of particles near the bottom and sides of the channel are deflected frequently. Even though the channel is smoothly paved with brick, a portion of the moving particles meets obstructions at the edges of the paving, and are deflected upward from the bottom and inward from the sides, and thus lose their onward velocity that they tend to take from gravity in the general direction.

Now, in the cases of water flowing in pipes, there are projections on the sides of the pipes that influence and retard the *onward* motion of the particles.

We cannot so readily observe the influences of obstructions in pipes as in channels, but Mr. Darrach has given us some very valuable facts relating to them, in his columns of *co-efficients*, which are the measures of the influences of those obstructions.

If there are in the pipes any incrustations, or if the pipe castings are not smooth (and, in the ordinary cases, they are very far from being smooth—with the microscope they appear very rough) thousands and tens of thousands of these molecules or particles of water strike the little projectiles that are scarcely visible to the naked eye, and lose, by a serious of reactions and deflections, a portion of the velocity that they tended to take from the inclination of the pipe or from the equivalent head pressure of water. Since the co-efficients in the formula for flow presented are variable for different conditions of velocity and diameter and smoothness of pipes, and since they can be determined only by practical experiment, the series given to us by Mr. Darrach will prove exceedingly valuable to those who have to place pipe systems on a large scale.